

Calculation of energy of an electron:-

The total energy of an electron revolving in a particular orbit is calculated by adding its potential energy & kinetic energy.

$$\begin{aligned} \text{Total Energy} &= P.E. + K.E. \\ \text{The kinetic energy of the electron} &= \frac{1}{2} m u^2 \\ \text{Potential energy} &= -\frac{k Z e^2}{r} \end{aligned}$$

$$\begin{aligned} \text{Hence Total Energy} &= P.E. + K.E. \\ &= -\frac{k Z e^2}{r} + \frac{1}{2} m u^2 \quad \text{--- (1)} \end{aligned}$$

We know that: the centrifugal force is equal to Coulomb's attraction force

$$\begin{aligned} \text{So,} \quad \frac{m u^2}{r} &= \frac{k Z e^2}{r^2} \\ \text{or } m u^2 &= \frac{k \cdot Z e^2}{r} \quad \text{--- (2)} \end{aligned}$$

On substituting the value of $m u^2 = \frac{k Z e^2}{r}$ in equation (1)

$$\begin{aligned} \text{Hence,} \quad E_{\text{Total}} &= -\frac{k Z e^2}{r} + \frac{k \cdot Z e^2}{2r} \\ \text{or } E_{\text{Total}} &= \frac{k Z e^2}{2r} - \frac{k Z e^2}{r} = \frac{k \cdot Z e^2}{r} \left(\frac{1}{2} - 1 \right) \\ &= \frac{k \cdot Z e^2}{r} \left(\frac{1-2}{2} \right) \\ &= -\frac{k \cdot Z e^2}{2r} \quad \text{--- (3)} \end{aligned}$$

Again substituting the value of $r = \frac{\sigma^2 h^2}{4 \pi^2 m \cdot Z e^2 \cdot k}$ in equation (3)

$$\begin{aligned} E_{\text{Total}} &= -\frac{k \cdot Z e^2}{2} \times \frac{4 \pi^2 m Z e^2 \cdot k}{\sigma^2 h^2} \\ &= -\frac{2 \pi^2 Z^2 e^4 \cdot m \cdot k^2}{\sigma^2 h^2} \quad \text{--- (4)} \end{aligned}$$

Thus the total energy of electron in an orbit of hydrogen atom is given by

$$E_n = -\frac{2 \pi^2 Z^2 e^4 m k^2}{\sigma^2 h^2}$$

where E_n is the total energy of electron moving in an orbit.

Calculation after putting the values.

$\pi = 3.1416$, $k = 9 \times 10^9 \text{ Nm}^2/\text{e}^2$, $e = 1.6 \times 10^{-19} \text{ C}$, $m = 9.1085 \times 10^{-31} \text{ kg}$
and $h = 6.6252 \times 10^{-34} \text{ joules-sec}$.

$$E = -\frac{21.79 \times 10^{-19}}{\sigma^2} \text{ joules atom}^{-1}$$

$$\begin{aligned} \text{or } E &= -\frac{313.5208}{\sigma^2} \text{ K.Cal mol}^{-1} \\ &= \frac{21.79 \times 10^{-19}}{\sigma^2} \times 6.2419 \times 10^{18} \text{ eV} \quad [1 \text{ joule} = 6.2419 \times 10^{18} \text{ eV}] \\ &= -\frac{13.6}{\sigma^2} \text{ eV} = -\frac{13.6}{\sigma^2} \times 23.053 \text{ KCal} \quad [1 \text{ eV} = 23.053 \text{ K.Cal}] \end{aligned}$$

where $\sigma = 1, 2, 3 \dots$

Calculation of energy of an electron:-

The total energy of an electron revolving in a particular orbit is calculated by adding its potential energy & kinetic energy.

kinetic energy of the electron = $\frac{1}{2} m u^2$
 Potential energy = $-k Z e^2 / r$

Hence Total Energy = P.E + K.E
 $= -\frac{k Z e^2}{r} + \frac{1}{2} m u^2$ — (11)

We know that: the centrifugal force is equal to Coulombic attraction force

so, $\frac{m u^2}{r} = \frac{k Z e^2}{r^2}$

or $m u^2 = \frac{k \cdot Z e^2}{r}$ — (12)

On substituting the value of $m u^2 = \frac{k Z e^2}{r}$ in equation (11)

we have, $E_{Total} = -\frac{k Z e^2}{r} + \frac{k \cdot Z e^2}{2r}$

or $E_{Total} = \frac{k Z e^2}{2r} - \frac{k Z e^2}{r} = \frac{k \cdot Z e^2}{r} \left(\frac{1}{2} - 1 \right)$
 $= \frac{k \cdot Z e^2}{r} \left(\frac{1-2}{2} \right)$
 $= -\frac{k \cdot Z e^2}{2r}$ — (13)

Again substituting the value of $r = \frac{n^2 h^2}{4 \pi^2 m \cdot Z e^2 \cdot k}$ in equation (13)

$E_{Total} = -\frac{k \cdot Z e^2}{2} \times \frac{4 \pi^2 \cdot m \cdot Z e^2 \cdot k}{n^2 h^2}$
 $= -\frac{2 \pi^2 Z^2 e^4 \cdot m \cdot k^2}{n^2 h^2}$ — (14)

Thus the total energy of electron in an orbit of hydrogen atom is given by

$E_n = -\frac{2 \pi^2 Z^2 e^4 m k^2}{n^2 h^2}$

where E_n is the total energy of electron moving in an orbit.

Calculation after putting the values:

$\pi = 3.1416$, $k = 9 \times 10^9 \text{ Nm}^2/\text{C}^2$, $e = 1.6 \times 10^{-19} \text{ C}$, $m = 9.1085 \times 10^{-31} \text{ kg}$
 and $h = 6.6252 \times 10^{-34} \text{ joules-sec}$

$E = -\frac{21.79 \times 10^{-19}}{n^2} \text{ joules atom}^{-1}$

$= \frac{21.79 \times 10^{-19}}{n^2} \times 6.2419 \times 10^{18} \text{ eV}$ [1 joule = $6.2419 \times 10^{18} \text{ eV}$]
 $= -\frac{13.6}{n^2} \text{ eV} = -\frac{13.6}{n^2} \times 23.053 \text{ kcal}$ [1 eV = 23.053 kcal]

or $E = -\frac{313.5208}{n^2} \text{ K.Cal mol}^{-1}$

where $n = 1, 2, 3 \dots$